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The physical assumptions and mathematical framework are given of a simplified, 2-dimensional model of a hydrodynamic disturbance due to passage of a submerged submarine in an ocean characterized by a thin thermocline. Several exploratory applications of the model are described, including estimation of: currents at the surface due to i) the complex disturbances made by a submarine running slightly "heavy" above a thin thermocline, and ii) to an oscillating and eventually collapsing wake just below a thin thermocline, and of the energetic efficiency of production of the lowest mode internal wave of a thin thermocline due to wake collapse.

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TECHNICAL REPORT 231-34

APPLICATIONS OF A TWO-DIMENSIONAL  
MODEL OF SUBMARINE DISTURBANCE  
IN AN OCEAN WITH A  
THIN THERMOCLINE

By

K. K. Wong, Peter Van Dyke  
and Sidney G. Reed

November 1972

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ABSTRACT

The physical assumptions and mathematical framework are given of a simplified, 2-dimensional model of a hydrodynamic disturbance due to passage of a submerged submarine in an ocean characterized by a thin thermocline. Several exploratory applications of the model are described, including estimation of: currents at the surface due to i) the complex disturbances made by a submarine running slightly "heavy" above a thin thermocline, and ii) to an oscillating and eventually collapsing wake just below a thin thermocline, and of the energetic efficiency of production of the lowest mode internal wave of a thin thermocline due to wake collapse.

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NOTATION

- A amplitude of hull disturbance =  $A \sin \gamma t$
- $a_0$  hull radius, or initial wake radius
- a radius of equivalent fully mixed wake
- b thermocline thickness
- $\beta$  =  $x/t = \text{const.}$  locus of stationary phase
- d bottom depth
- $\delta$  delta function
- $\epsilon$  =  $\frac{\rho_2 - \rho_1}{\rho_2}$  relative density jump across thermocline
- f source depth below thermocline
- g acceleration of gravity
- $\gamma$  frequency factor in representation of hull disturbance  
as  $A \sin \gamma t$
- H Heaviside unit function
- h thermocline depth
- $x$  =  $\beta s - \rho(s)$
- L hull length
- M strength of moving source representing final collapse  
stage =  $M/t$
- N local Vaisala frequency
- $\nu$  frequency factor in time-dependent quadrupole strength  
 $Bte^{-\nu t}$

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- $\psi(\tau)$  time-dependent source function
- $\rho_1$  density of upper layer
- $\rho_2$  density of lower layer
- $s$  wave number, variable of integration
- $s_o$  root of  $d\sigma/ds = \beta$
- $\sigma$  factor in integral defining velocity due to unit move,  
Equations [1] and [2]
- $\sigma_o$  value of  $\sigma(s)$  for  $s = s_o$
- $t$  time
- $T$  interfacial Vaisala period =  $2\pi/N$
- $T_1 = 2\pi/\gamma$  duration of source representing hull disturbance,  
Equation [4]
- $\tau$  source time parameter
- $\theta$  factor in integral defining velocity response to unit  
source, Equations [1] and [2]
- $u(x, t; \xi, \tau)$  velocity at surface due to unit source at  
( $\xi, -f, \tau$ )
- $U(x, h, t)$  velocity at surface, at time  $t$  due to time  
dependent source function  $\psi(\tau)$
- $U_1$  integrated velocity at surface due to hull, Equation [4]
- $U_2$  integrated velocity at surface due to wake collapse,  
Equations [7] and [8]
- $\bar{U}_2$  secular (non-propagating) part of  $U_2$ , Equation [7]

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$\underline{\underline{U}}_2$	wavelike (propagating) part of $U_2$ , Equation [8]
$U_3$	integrated velocity at surface due to final phase of wake collapse
$\overline{U}_3$	secular part of $U_3$ , Equation [9]
$\underline{\overline{U}}_3$	wavelike part of $U_3$ , Equation [10]
$V_o$	hull velocity
$W$	source speed for final stage of collapse
$\xi$	x coordinate of source
$\overline{x}$	coordinate along axis of track of submarine

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## INTRODUCTION

For several years, use has been made at HYDRONAUTICS, Incorporated of a simplified model of hydrodynamic disturbances simulating those that would be produced by passage of a submerged submarine in ocean conditions characterized by a thin thermocline (more accurately, pycnocline). A brief description of some early results obtained with this model were given by M. P. Tulin in Reference 1. Recently, more sophisticated models have been introduced with which analytical estimates and numerical calculations have been made of hydrodynamic effects due to internal waves excited by translational movement of the submerged hull and collapse of the submarine wake. Recognizing that these models can give more accurate results, we have continued to make use of our more simplified model for exploratory purposes, to obtain orders of magnitude of effects such as surface currents and strains due to sources to which less attention has been paid, such as: wakes with net vertical momentum; and to combinations of this with the complex array of sources simulating the sequence of hydrodynamic disturbance associated with passage of a submerged submarine. It is hoped that the results will be sufficiently suggestive to stimulate further work along these lines with more accurate models.

Given below are: 1) a description of the physical assumptions of the model; 2) a brief outline of its mathematical development; 3) comparison of some of its results with experimental data and with other theoretical results for the case of a thin thermocline; 4) results of applications of the model to the case of a submarine moving "heavy" in the mixed region above

the thermocline, and moving "light" under the thermocline; and to the (energetic) efficiency of wake collapse as a source of lowest mode internal waves; and 5) a conclusion, including some remarks regarding limitations of the model, and in regard to dependence of its results on operational and environmental parameters.

### 1. PHYSICAL ASSUMPTIONS

The model embodies several simplifications. First, it is two-dimensional, and may be valid to the extent that a "slender body" approximation holds, i.e., the flow parameters change only gradually along the direction of hull motion and wake length, and relatively rapidly in the plane perpendicular to this direction. For wake phenomena at all but low speeds, such an approximation may be valid. For disturbances due to the hull itself, the approximation may be less appropriate.

Point source hydrodynamic disturbances are assumed, with strengths varying with time corresponding, approximately, to phenomenological features of the actual sources. Thus the disturbance due to a (cylindrically symmetric) hull of radius  $a_0$  and length L moving with velocity  $V_0$  is represented by a two-dimensional monopole proportional to  $\sin \gamma t$  where  $\gamma = 2\pi V_0 / L$ ; the early stages of collapse of a wake of radius a in a density gradient with Vaisala frequency N is represented by a point quadrupole of strength proportional to  $t e^{-vt}$  where  $v \sim N$  is the frequency corresponding to the maximum of the collapse spectrum (see Wu, Reference 2); and the wake with initial vertical momentum  $M_0$  is assumed to be a dipole of constant strength, initially proportional to  $M_0$ , with a moving center. The use of

such point sources implies that hydrodynamic disturbances are not expected to be accurately represented at short distances from the source; and, inversely, that the actual source size can be supposed small compared to a) the scale over which the density gradient can be considered constant, and b) the (vertical) internal wave length.

As mentioned above, the ocean environmental conditions are supposed to be those in which a thin thermocline occurs. This is further simplified, in the model, to a two layer fluid with the effects of the actual (assumed weak) density gradient below the thermocline is assumed to be incorporated entirely in the source strengths, and, for moving wakes, in geometrical features of the trajectory. Thus, for example, the strength of the quadrupole representing the collapsing wake will depend on the local Vaisala frequency  $N$ , and on features of its geometry and internal turbulence, but the disturbance due to it is assumed to be equivalent to that of a quadrupole of the same strength and geometric disposition in a two layer fluid. There is supposed to be no effect on source strength or motion of the deformation of the density interface, which will generally consist of a local disturbance and interfacial waves radiated away. The latter simulate only the lowest, "heaving" mode of a thin thermocline; the higher internal wave modes which would be present in the actual density gradient below the thermocline are simply neglected.

## 2. MATHEMATICAL FRAMEWORK

Across the density interface, which is supposed to undergo small displacements, continuity of velocity and pressure are assumed. The origin of coordinates is at the interface with the

axis along the interface. The upper layer, of density  $\rho_1$ , has its upper boundary at  $y = h$ , taken to be a rigid wall; the bottom of the lower layer, of density  $\rho_2$ , is at  $z = -d$ . The source of disturbance is at a distance,  $\pm f$  from the interface. The medium is supposed to be at rest for time  $t = 0$ . Fourier transform techniques are used to find solutions to the equations of motion and boundary conditions, for a unit "influence function" source at  $x = \xi$ ,  $y = \pm f$  and  $t = \tau$ . Thus, for example, the velocity at  $x, h, t$  due to a unit source at  $(\xi, -f, \tau)$  is

$$u(x, t; \xi, \tau) = - \frac{\delta(t-\tau)}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \theta s \sin(s(x-\xi)) ds + \frac{H(t-\tau)}{\sqrt{2\pi}} \int_{-\infty}^{\infty} s \sigma \theta \sin(\sigma(t-\tau)) \sin(s(x-\xi)) ds \quad [1]$$

where  $\delta$  and  $H$  are the delta and heaviside unit functions, and

$$\theta = - \frac{2\pi \cosh(|s|(d-f))}{|s| \sinh(|s|(h+a))}$$

$$\sigma^2 = \frac{g|s|\epsilon \sinh(|s|h) \sinh(|s|d)}{\sinh(|s|(a+h))} \quad [2]$$

For a source function having prescribed time dependence  $\varphi(\tau)$  the velocity  $U$  at the wall is

$$U(x, h, t) = \int u(x, t; \xi, \tau) \varphi(\tau) d\tau \quad [3]$$

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The results for (vertical) dipoles or quadrupoles are obtained by differentiating with respect to  $f$ . Stationary phase approximations can be found in the usual way, when periodic factors can be isolated in the integrals to obtain asymptotic relations for large  $t$ . For other cases numerical procedures have been used.

Thus for a half-cycle harmonic source at  $(0, -f)$  whose strength varies with time  $> 0$  according to  $A \sin \gamma\tau$ , and is 0 for  $\tau > T_1$ , where  $\gamma = 2\pi/T_1$ ,

$$U_1(x, t) = \int_0^{T_1} A \sin \gamma \cdot \tau u(x, t; 0, \tau) d\tau \quad [4]$$

where, for  $t > T_1$

$$u(x, t; 0, \tau) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} s \sigma^a \sin(\sigma(t-\tau)) \sin sx ds \quad [5]$$

The asymptotic formula for  $U_1$  is for  $x \geq 0$ ,  $x/t = \beta = \text{constant}$ ,  $(\rho_2 - \rho_1)/\rho_2 = \epsilon \ll 1$  and large  $t$ ,

$$U_1(\beta, t) \approx -2A \sqrt{\frac{2\pi}{t}} \frac{e^{-s_o(f+h)}}{|x''_o|^{\frac{1}{2}}} \frac{\gamma \sigma_o}{\gamma^2 - \sigma_o^2} \sin \frac{\sigma_o T_1}{2} \sin(x_o t + \frac{\sigma_o T}{2} + \frac{\pi}{4}) \quad [6]$$

where  $x(s) = \beta s - \sigma(s)$ , and  $x''_o$ ,  $s_o$ ,  $\sigma_o$  denote, respectively, the value of  $d^2x/ds^2$ ,  $s$  and  $\sigma$  evaluated at  $s_o$ , the root of  $d\sigma/ds = \beta$ .

Similarly, the velocity at the upper boundary due to a quadrupole at  $(0, -f)$  whose strength varies with time as  $Bte^{-\nu t}$  is

$$U_2(x, t) = \bar{U}_2 + \bar{\bar{U}}_2$$

where

$$\bar{U}_2 = - \frac{4Bte^{-\nu t} \times [3(f+h)^2 - 3x^2]}{[(f+h)^2 + x^2]^3} \quad [7]$$

and

$$\begin{aligned} \bar{\bar{U}}_2 &= 2B \int_0^\infty e^{-s(f+h)} \frac{\sigma_s^2}{(\gamma^2 + \sigma^2)^2} \frac{\gamma^2 - \sigma^2}{2} (\cos(sx - \sigma t) - \cos(sx + \sigma t)) \\ &\quad - \gamma\sigma [\sin(sx - \sigma t) + \sin(sx + \sigma t)] ds . \end{aligned} \quad [8]$$

As mentioned above, this result has been used to model the early stage of wake collapse.

For the final stage of wake collapse, two sources and two sinks are supposed to have been concentrated at  $(0, -f)$  at  $t = 0$ . As time increases, the two sources move outwards, in opposite directions with constant velocity  $W$ , while their strengths vary as  $M/t$  where  $M = \text{constant}$ . The two sinks spread out to form a line sink of uniform strength  $-V/Wt^2$  joining the two sources, (Reference 3). For this case the velocity induced at the surface  $U_3 = \bar{U}_3 + \bar{\bar{U}}_3$

where

$$\bar{U}_3 = \frac{2M}{t} \left[ \frac{x-Wt}{(f+h)^2 + (x-Wt)^2} + \frac{x+Wt}{(f+h)^2 + (x-Wt)^2} \right] - \frac{M}{Wt^2} \ln \frac{(f+h)^2 + (x+Wt)^2}{(f+h)^2 + (x-Wt)^2} \quad [9]$$

is a secular term, and

$$\bar{\bar{U}}_3 = 4M \int_0^t \frac{d\tau}{\tau} \int_0^\infty e^{-s(f+h)} \sigma \sin s x \sin (\sigma(t-\tau)) \left[ \frac{\sin sW\tau}{sWt} - \cos sW\tau \right] ds \quad [10]$$

### 3. COMPARISON WITH EXPERIMENT

Comparison was made with results of two-dimensional experiments by Wu (Reference 2) in which measurements were made of interfacial and internal waves and velocities at the surface due to collapse of a premixed mass of fluid. The density profile in these experiments involved a sharp, variable density jump above a constant linear density gradient; the collapse occurred at a fixed location relative to the density interface. Three different values of density jump were used. Wu's observations indicated that for the largest density jump, interfacial waves clearly predominated; for the intermediate value of density jump both interfacial and higher mode internal waves were present; and for the smallest jump, the higher mode internal waves predominated. It was felt that some comparison should be possible

with his results for the case of the largest density jump. For this purpose we have used the asymptotic approximation and assumed a quadrupole source to represent the collapse with strength varying at  $Bte^{-vt}$  where  $B = N^2 a^4$ ,  $a$  = radius of mixed region.

From the slopes of the rays along which wave energy was concentrated, some estimates can be made of the frequency distribution of the source associated with the wake collapse. For the case in which interfacial waves predominated, the value of  $v$  was estimated in this way to be  $v \sim 0.8N$ . The value of  $N$  in the experiments was about  $0.7 \text{ sec}^{-1}$ . We then find  $(U_2 / \sqrt{geh})_{\max} \sim 0.03$  to compare with Wu's observed  $0.04$  (corresponding to Run 4 on p. 33 of Reference 2).

The agreement is probably somewhat fortuitous owing to the fact that the quadrupole strength is only approximate. However, the maximum  $U_2 / \sqrt{geh}$  does not seem sensitive to exact values of the parameters involved.

If the same procedure is followed for Wu's case 2, with an intermediate density jump, the agreement is poorer. Wu found a maximum  $U_2 / \sqrt{geh}$  about four times larger, but above expression gives a smaller value than for the previous case. This discrepancy can be ascribed to the presence of the higher mode internal waves observed by Wu, and which are not treated by this theory.

### 3.a Comparison with Miles' Theory for Sharp Thermocline Case

Comparison can be made with the theoretical results of the three-dimensional calculations by J Miles (Reference 4) for the case of a thin thermocline. In a plane perpendicular to the direction of motion, the two-dimensional model of the hull dis-

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turbance corresponds to flow induced by a time-dependent monopole of strength:  $A \sin \gamma t$  where  $A \sim a_o^2 V/L$  in which  $a_o$  = radius and L the length of the (assumed axisymmetric) hull,  $V_o$  the (constant) forward speed, and  $\gamma = 2\pi/T$ , with  $T = L/V_o$ .

For the lowest internal wave mode of a thin thermocline, the asymptotic two-dimensional result for the surface strain  $I_1$  due to the hull at depth  $f + h$  is given by Equation [6] above. Omitting the periodic dependence on  $t$ , and following the same approximation procedure as Miles', the maximum  $I_1$  is found to occur, approximately, at  $s_o \sim \frac{9}{4} \frac{l}{f+h}$  for which (replacing  $t$  by  $\underline{x}/V_o$ )

$$|I_1|_{\max} \sim 0.4 a_o^2 L (\underline{x} V_o)^{-\frac{1}{2}} (g\epsilon)^{\frac{1}{4}} (f+h)^{-9/4} \quad [11]$$

Except for notation, Equation [11] is the same as Miles' result for the hull disturbance (Reference 4).

The asymptotic expression for the surface strain induced by a collapsing wake, modeled by a two-dimensional quadrupole in a transverse plane, with the time dependence  $Bte^{-vt}$  is

$$|I_2| \sim 2B \sqrt{\frac{2\pi}{t}} \frac{e^{-s_o(f+h)}}{|x_o''|^{\frac{1}{2}}} \frac{s_o^3}{(\gamma^2 + \sigma_o^2)^2} \left| \left( -\frac{\gamma^2 - \sigma_o^2}{2} \cos(x_o t + \frac{\pi}{4}) + \gamma \sigma_o \sin(x_o t + \frac{\pi}{4}) \right) \right| \quad [12]$$

the maximum  $I_2$  is found to occur at about  $s_o = \frac{11}{4} \frac{l}{f+h}$ . With  $B \sim N^2 a^4$ , where  $N^2$  is the Vaisala frequency at depth  $f + h$  and  $a$  is the effective mixed wake radius at collapse:

$$|I_2|_{\max} \sim 12N^2a^4 \left(\frac{V_o}{\bar{x}}\right)^{\frac{1}{2}} (f+h)^{-11/4} (g\epsilon)^{-5/4} \quad [13]$$

Except for notation and the numerical factor in front being an order of magnitude larger, Equation [13] is the same as Miles' result for the collapsing wake. Despite the uncertainty in source functions, this discrepancy may reflect the fact that the two-dimensional approximation tends to overestimate and on the other, Miles' use of a small-body type approximation tends to under-estimate the surface disturbances due to wake collapse.

#### 4. APPLICATIONS OF MODEL

This model has been employed to investigate effects due to a submarine in the upper mixed layer, above a thin thermocline, but running out of trim, and "heavy". Its wake will then have a negative momentum in the vertical direction, moving down toward the thermocline. The wake will entrain water on its way down, and slow down; however, we have assumed, for simplicity, a constant vertical dipole moment and constant velocity for the wake until it reaches the thermocline. Actually, the velocity is inversely proportional to the distance traveled by the wake, divided by the initial effective wake radius (Reference 5); under the geometrical circumstances assumed the velocity probably would not vary by as much as 50 percent. When the wake encounters the thermocline, it is assumed the downward motion is arrested, simulated by the addition of a negative dipole moment, after which the dipole moment is assumed to undergo a cycle of damped oscillation to zero, similar to phenomena noted in Reference 5.

Before all this, one cycle of a two-dimensional monopole is assumed to occur, representing the effect of the hull. The geometrical arrangement and time-history of these sources is depicted in Figure 1. The hydrodynamic source parameters were: representing the hull, a monopole source  $= A \sin \gamma t$  with  $A \sim a_0^2 V_0 / L$  and  $\gamma \sim V_0 / L$ , with  $V_0 \sim 6$  knots; radius  $a_0 = 10^3$  cm, somewhat larger than usual, and length to radius ratio  $L/a_0 \sim 15$ . The depth is 20 meters above the thermocline, which is at 60 meters. Across the thermocline the relative change in density  $\Delta\rho/\rho = 3 \cdot 10^{-4}$ . The wake dipole moment has the initial value  $D_1 \sim 5a_0^2$ , corresponding to a few tons "heavy".  $D_1$  is assumed to remain constant, with its center moving downward at a constant speed 5 cm/sec, until within  $a_0$  cm of the interface, when  $D_1$  is switched off by adding  $D_2 = -D_1$ .  $D_3$  is then switched on, representing the overshoot and fall-back of the wake by the damped oscillatory function  $D_3 (\cos \alpha t) e^{-\alpha t}$  with  $\alpha \sim 3/T$ ,  $T = (g\epsilon/h)^{-\frac{1}{2}} \approx 140$  sec is the "Vaisala" period of the interface itself.

Figure 2 shows the behavior of the velocity at the wall for each of the sources of Figure 1, separately, and for the superposition of all sources together, for two multiples of interface Vaisala periods  $T$ ,  $2.8T$  and  $5.6T$ , corresponding roughly to 400 and 800 sec. Evidently the sum of all the contributions to the velocity at the wall exhibits effects of some "destructive interference", being smaller than the contributions for most of the individual sources. None of the individual sources is negligible compared to the rest. Undoubtedly the "destructive interference" reflects the particular assumptions involved in this calculation. The time phasing assumed depends on the

assumed trajectory of the downward moving dipole. Actually, it would be expected that the wake trajectory will differ from that assumed, due to the entrainment, and would slow down as it moved toward the thermocline (Reference 5). Also the effective radius of the wake will grow, so that it will reach the thermocline somewhat sooner than assumed; the dipole effects, therefore, may be somewhat exaggerated. Calculations taking into account these adjustments have not been carried out.

Corresponding to the situation in which a submarine is running slightly "light" below a thin thermocline, Figure 3 shows the time sequence assumed for wake with upward momentum and wake collapse. The wake is assumed to have a damped upwards trajectory oscillating about the same level at which it collapses (Reference 6), ten meters below the density interface, which is at a depth of 50 meters. Shallow conditions are assumed, with the bottom at 150 meters. The source parameters correspond to a wake radius  $a_0 = 10^3$  cm, and a maximum upwards velocity of 3 cm/sec, corresponding to a dipole moment  $D = 3.10^6 (\cos \alpha t) e^{-\alpha t}$ , with  $\alpha = 1/4T$ , where  $T =$  the interface Vaisala period, again about 140 sec. The quadrupole strength is taken to be  $Q = Bte^{-\gamma t}$ , with  $B \sim \frac{1}{2}N^2a_0^4$ , where now  $N$  is the Vaisala frequency of the actual density gradient below the thermocline and  $2\pi/N \sim 4T$ ,  $\gamma = 1/4T$ . The velocities at the surface due to the dipole and quadrupole separately and superposed are shown in Figure 4 as a function of distance at  $t = 2\pi T \sim 816$  sec. In this case the quadrupole appears to have a larger effect, by about a factor 2, and the individual contributions reinforce each other.

The relative separation in time of these sources shown in Figure 3 may be larger than actually is the case. Tulin has observed that the collapse often begins at the same instant as the wake begins to fall (Reference 6). If this would be the case, the quadrupole effects would be larger. Laboratory experiments reveal a complex behavior of the terminal phase of trajectories of turbulent vortex pairs, simulating wakes with vertical momentum in a density gradient; and the final wake collapse, in such cases, appears to be a more complicated function than shown in Figure 3 (Reference 5). Calculations with different values of relative source phasing have not been carried out.

It may be worth noting also that the results shown in Figures 2 and 4 are derived from numerical evaluation of integral expressions for the full disturbance, and not only the asymptotic approximations.

The model has been employed also to explore the effect of different ratios of bottom depths to thermocline depths. The bottom effects on disturbances due to the hull are generally small except for quite shallow depths. Such effects for the quadrupole, for the same assumed conditions, are generally smaller.

Another problem to which this model has been applied is that of the "efficiency" of internal wave production by a collapsing wake. Efficiency is here defined as the ratio of the energy of the lowest mode internal wave, long after collapse, to the initial potential energy of the collapsing mass of fluid, assumed fully mixed and initially quiescent. For such a fixed time, the energy of the lowest mode internal wave was estimated as

$$2 \int_0^{\infty} \Delta\rho y^2 g dx; \text{ where } \Delta\rho \text{ is the density jump across the thin}$$

thermocline and  $y$  is the thermocline displacement. The time dependence of the collapsing wake was again taken  $\sim Bte^{-vt}$ : where  $B$  was taken to be  $N^2 a^4$  where  $N$  is local Vaisala frequency and  $a$  the effective wake radius. Total potential energy of the wake was taken to be  $\pi/4 N^2 a^4$ . The result for Wu's experiments was about 6 percent, which seemed surprisingly small. A check on this was made using Wu's experimental values for interface displacement to evaluate the above integral, and gave nearly the same result. Lighthill had remarked that the efficiency of lowest mode generation by a collapsing wake would be small (Reference 7; see also Reference 9).

Some exploration was also conducted of effects of different time-dependencies of the quadrupole source. First by using the same functional form, i.e., strength proportional to  $t e^{-vt}$ , and calculating surface strains and currents for fixed source geometry and values of density jump across the thermocline, for a range of values of  $v$ . Some appreciable variation of the was found. Next calculations were made for a more general functional form  $t^n e^{-vt}$ , where  $n$  was allowed to be  $<1$  and  $>1$ . This investigation could not be completed.

It may be noted that the secular, local disturbance velocity induced by a quadrupole of strength  $Bte^{-vt}$  located at depth  $f+h$  below a thermocline if depth  $h$  is given by  $\bar{U}_2$  from Equation [7] above. Near  $x = 0$ , the surface strain due to wake collapse, modeled by this source, approaches, for long times

$$\overline{I}_2 \rightarrow - 12B/v^2 (f+h)^4$$

or, with  $B \sim N^2 a^4$ ,  $I_2 \rightarrow 12N^2 a^4/v^2 (f+h)^4$ ; with  $v \sim N$ , if the maximum effective wake radius  $a$  is of the order of 4 times the original wake radius  $a_0$ , Reference (8), quite appreciable values of  $I_2$  can result. A more accurate model of final wake collapse at large times would involve the source-sink combination discussed above, Equation [9]; this has not been carried out but it is believed that doing so would increase the value of  $I_2$  above.

##### 5. CONCLUDING REMARKS

It should perhaps be emphasized again that the results given above involve only the lowest mode internal wave of a thin thermocline. The fact that it appears to be the most persistent in usual ocean conditions lends importance to this mode. However, for thermoclines of thickness  $b > a_0$ , and submarine depths in the thermocline, the second and higher modes are likely to be more important. It would be desirable to extend this type of calculation to include these higher modes. Because of the differences in speeds of internal wave modes and of their dispersive characteristics, the results can be expected to be correspondingly more complicated.

Surface effects associated with the lowest mode of a thin thermocline have a dependence on depth dominated by the exponential in Equation [6] which can be regarded as the product of two factors: one a function of depth below the thermocline  $\exp(-s_{o_1} f)$  where  $s_{o_1}$  is the wave number of the lowest mode, and

the other,  $\exp(-s_{o_1} h)$ , of depth above the thermocline. The depth dependence of higher modes for a thick thermocline with a more or less well-defined upper edge at depth  $h$  will characteristically have an exponential dependence on  $h$ ,  $\exp(-s_{o_n} h)$  where  $s_{o_n}$  is the (larger) wave length of the  $n$ th mode, but a more or less periodic dependence on depth below the thermocline. Often the upper edge of a thick thermocline is not sharply defined, however, and  $h$  is then somewhat different for the various modes.

Speed dependence for the hull effects may be expected to increase at low speeds because of the "resonance" type denominator in Equation [6]. With a similar source, the same type of denominator may be expected to appear for every mode, with appropriately changed values of  $s_{o_n}$ . At higher speeds, the hull contribution decreases as in Equation [11]. The dipole strength will depend on speed through the vertical velocity, which is proportional to the forward speed, for the same angle of attack of the sailplanes or hull (Reference 1).

The quadrupole contribution increases with speed through the dependence on  $V^{\frac{1}{2}}$  explicit in Equation [13] and implicitly through dependence of effective wake size on speed. In a linear density gradient, rough upper bounds to this wake size can be obtained by assuming all the work of turbulent expansion of the wake from  $a_o$  to  $a$  goes into increasing potential energy. This gives  $(a/a_o)^4 \approx 1 + \frac{1}{4} \frac{V^2}{N^2 a_o^2}$  which seems to agree with most of the available laboratory data. However, for our model this can only apply for  $f \geq a$ . For smaller  $f$ , the wake encounters the thermocline during its growth, and the expansion beyond the thermocline must take

into account the nonlinearity of the overall density gradient. For higher modes in a thick thermocline, this type of speed dependence will hold for the modes having vertical wavelengths much larger than the size of the wake; this depends on the thickness of the thermocline. The modes with shorter wavelengths should not have such strong speed dependence (Reference 9).

Because of the propensity for internal waves to have nonlinear interactions, superposition of the internal wave disturbances, as assumed above, may exaggerate the surface effects so estimated.

The hydrodynamic source functions have been assumed to be "decoupled" from the internal wave field. Some basis for this assumption can be found in the fact that the trajectory of turbulent vortex pairs, and at least some aspects of wake collapse phenomena can be estimated without having to deal with the coupled phenomena (References 2 and 5). An attempt was begun to obtain effective (uncoupled) source strengths by systematic comparison of experimental data on interface deformations (References 2 and 5) with calculations involving the functional forms of sources assumed above, in order to obtain "best fit" parameters in these functions. Unfortunately, this work could not be completed.

It is felt nonetheless that the results indicate (1) that the effects of wake vertical momentum should be taken into consideration in the estimation of surface effects due to passage of a submerged submarine in a stratified ocean; and (2) that the superposition of the complex pattern of disturbances associated with actual submarine passage can give rise to significant

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variation in the magnitude of surface effects. It is recognized that these results are rather fragmentary; it is hoped, however, that they are sufficiently suggestive to stimulate further work with more accurate models, over wider ranges of operational and environmental parameters, and taking into account to the extent possible the coupling of the disturbance field to the dynamics of the wake.

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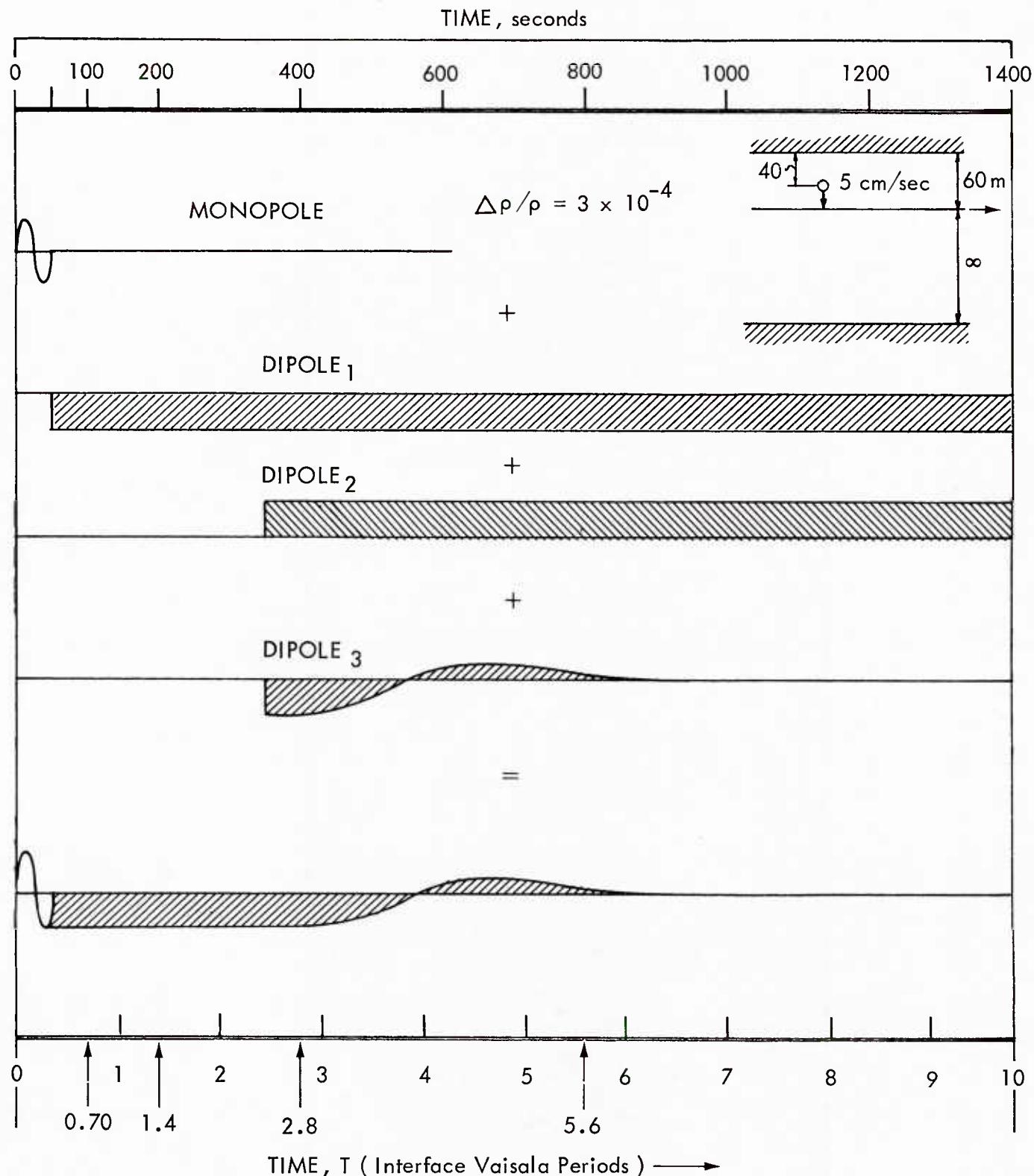


FIGURE 1 - TIME HISTORY OF HYDRODYNAMIC SOURCE CONTRIBUTIONS SIMULATING SUBMARINE RUNNING HEAVY ABOVE THERMOCLINE.

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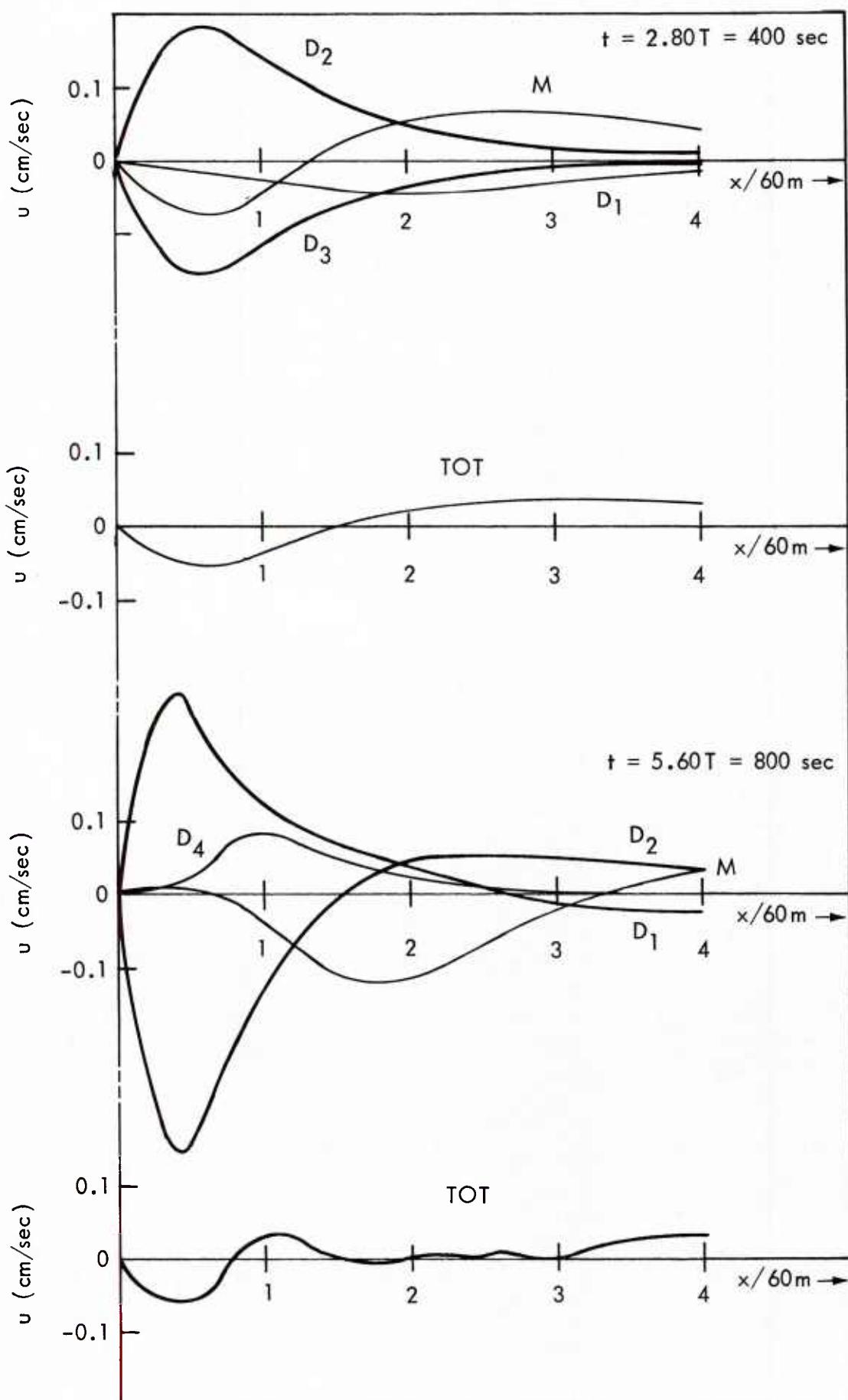


FIGURE 2 - DEPENDENCE ON DISTANCE TO SIDE OF VELOCITIES AT SURFACE DUE TO HYDRODYNAMIC SOURCES SHOWN IN FIGURE 1 AT 2.8 AND 5.6 INTERFACE VAI SALA PERIODS  $T$ .

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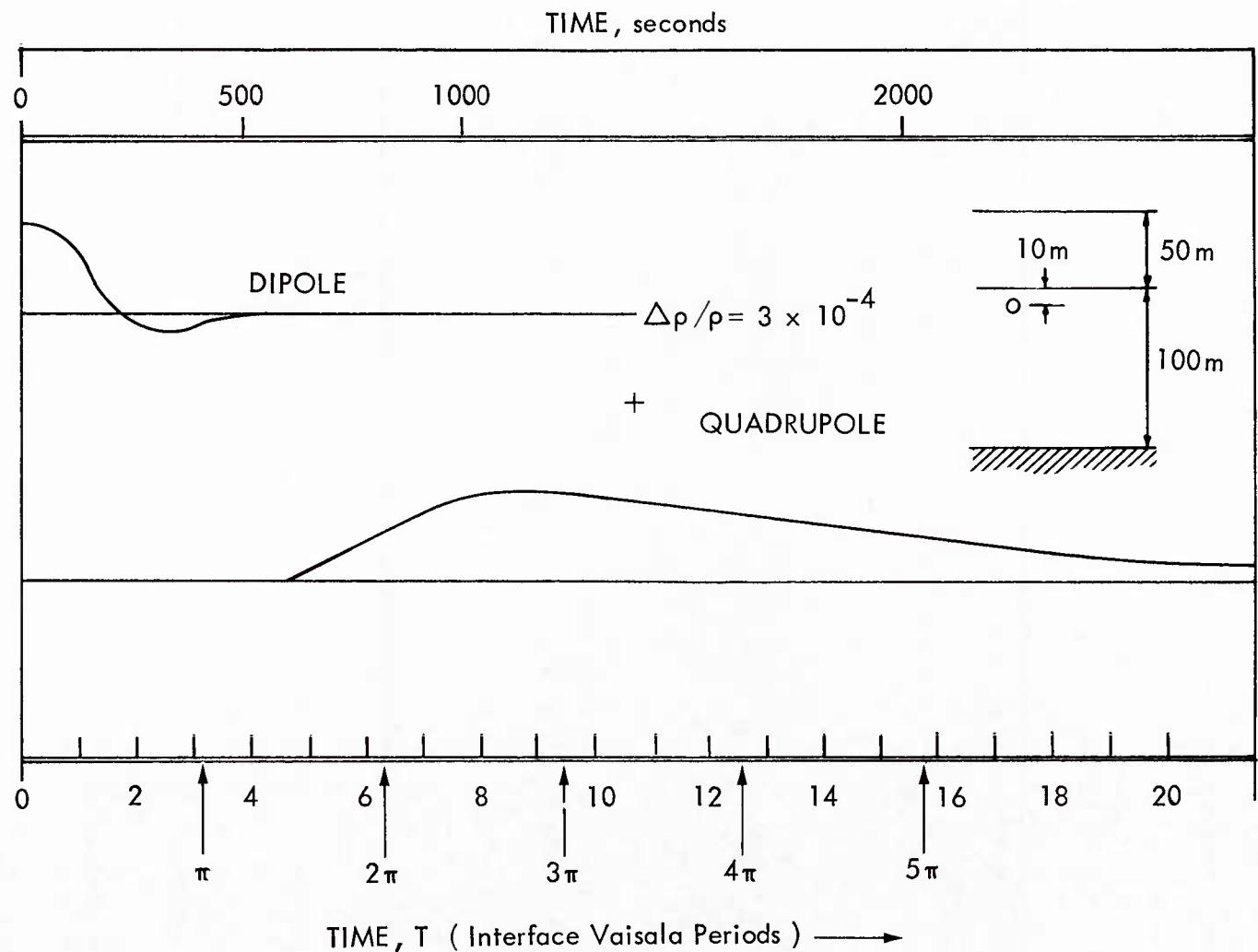


FIGURE 3 - TIME HISTORY OF HYDRODYNAMIC SOURCES SIMULATING WAKE OF SUBMARINE RUNNING LIGHT JUST BELOW THERMOCLINE.

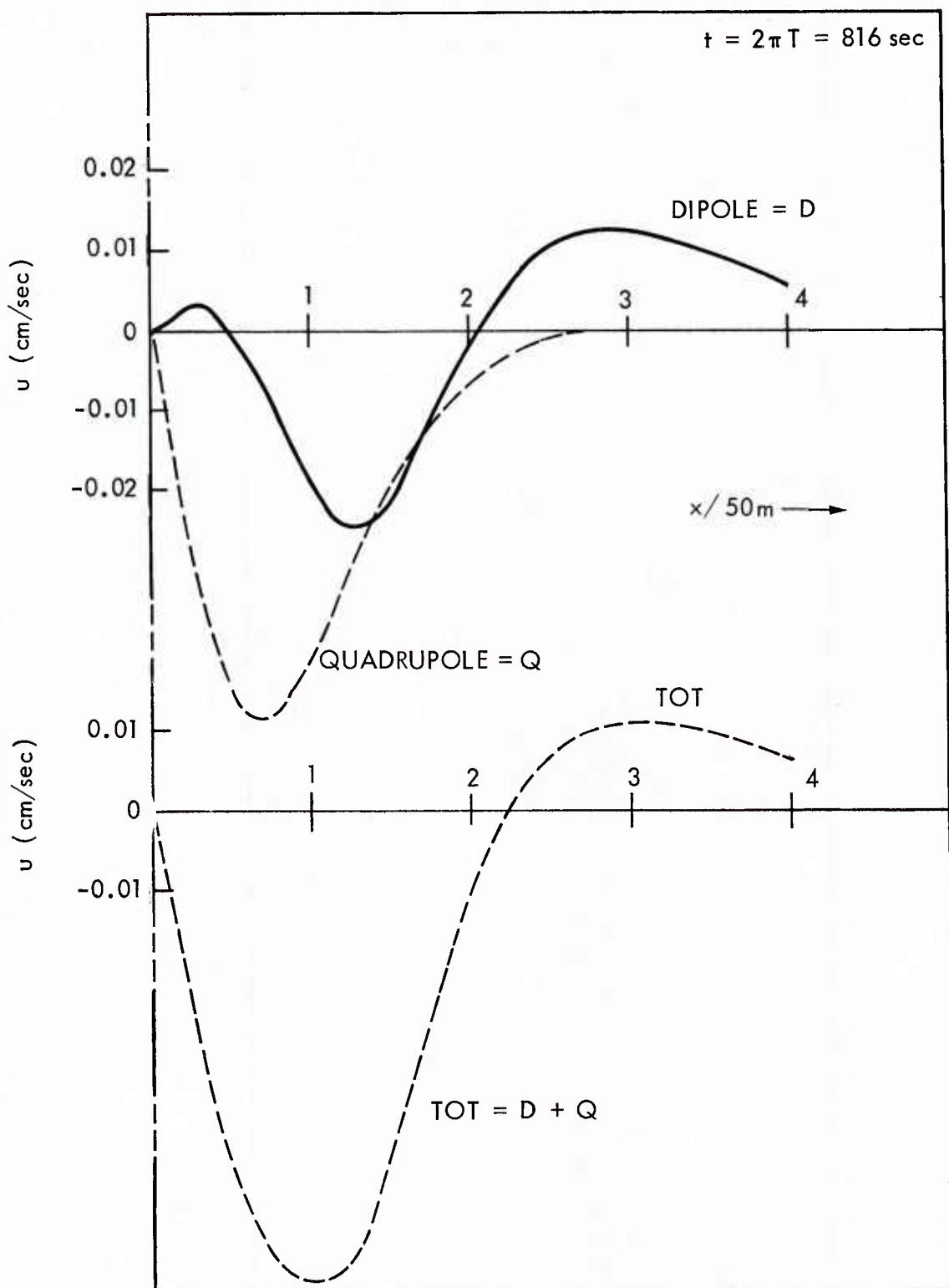


FIGURE 4 - VELOCITIES AT SURFACE DUE TO SOURCES IN FIGURE 3  
SIMULATING WAKE OF SUBMARINE SLIGHTLY LIGHT  
BELOW THERMOCLINE

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